

Possibility of cyclic Turnarounds In Brane-world Scenario: Phantom Energy Accretion onto Black Holes and its consequences.

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Abstract

A universe described by braneworlds is studied in a cyclic scenario. As expected such an oscillating universe will undergo turnarounds, whenever the phantom energy density reaches a critical value from either side. It is found that a universe described by RSII brane model will readily undergo oscillations if, either the brane tension, λ or the bulk cosmological constant, Λ_4 is negative. The DGP brane model does not readily undergo cyclic turnarounds. Hence for this model a modified equation is proposed to incorporate the cyclic nature. It is found that there is always a remanent mass of a black hole at the verge of a turnaround. Hence contrary to known results in literature, it is found that the destruction of black holes at the turnaround is completely out of question. Finally to alleviate, if not solve, the problem posed by the black holes, it is argued that the remanent masses of the black holes do not act as a serious defect of the model because of Hawking evaporation.

1 Introduction

Cyclic universe has always been a burning topic in the field of theoretical cosmology, since it is expected to avoid the initial singularity by providing an infinitely oscillating universe. However cyclic universe confront a serious problem of black holes(BHs). If the BHs formed during the expanding phase survives into the next cycle they will grow even larger from one cycle to the next and act as a serious defect in an otherwise nearly uniform universe. With the passage of time the BHs will occupy the entire horizon and then the cyclic models will break away. In this paper we investigate the possibility of an oscillating universe in two of the well known models of brane-world gravity, namely, RSII brane and DGP brane models.

Randall and Sundrum [1, 2] proposed a bulk-brane model to explain the higher dimensional theory, popularly known as RS II brane model. According to this model we live in a four dimensional world (called 3-brane, a domain wall) which is embedded in a 5D space time (bulk). All matter fields are confined in the brane whereas gravity can only propagate in the bulk. The consistency of this brane model with the expanding universe has given popularity to this model of late in the field of cosmology.

A simple and effective model of brane-gravity is the Dvali-Gabadadze-Porrati (DGP) braneworld model [3] which models our 4-dimensional world as a FRW brane embedded in a 5-dimensional Minkowski bulk. It explains the origin of dark energy(DE) as the gravity on the brane leaking to the bulk at large scale. On the 4-dimensional brane the action of gravity is proportional to M_p^2 whereas in the bulk it is proportional to the corresponding quantity in 5-dimensions. The model is then characterized by a cross over length scale $r_c = \frac{M_p^2}{2M_5^2}$ such that gravity is 4-dimensional theory at scales $a \ll r_c$ where matter behaves as pressureless dust, but gravity leaks out into the bulk at scales $a \gg r_c$ and matter approaches the behaviour of a cosmological constant. Moreover it has been shown that the standard Friedmann cosmology can be firmly embedded in DGP brane.

In the context of BHs and phantom energy accretion on BH, it should be mentioned that Jamil et al [4] studied charged BHs in phantom cosmology. Jamil in [5] has shown the evolution of a Schwarzschild Black Hole in Phantom-like Chaplygin gas Cosmologies. Primordial BHs in phantom cosmology and accretion of phantom DE on BTZ BHs were also studied by Jamil et al in [6, 7]. Nayak in [8] investigated the effect of Vacuum Energy on the evolution of primordial BHs in Einstein Gravity. Paolis in [9] studied BHs in bulk viscous cosmology. In the context of cyclic cosmology, it should be mentioned that Saridakis in [10] studied

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cyclic Universes from general collisionless Braneworld models. Cai et al in [11] investigated cyclic extension of the non-singular cosmology in a model of non-relativistic gravity. Cai et al in [12] investigated cyclic and singularity-free evolutions in a universe governed by Lagrange-multiplier modified gravity. Moreover Cai et al in [13] showed that gravity described by an arbitrary function of the torsion scalar, can provide a mechanism for realizing bouncing cosmologies, thereby avoiding the Big Bang singularity. Non-singular cyclic cosmology without phantom menace was also studied by Cai et al in [14].

We intend to study the effects and consequences of phantom energy accretion onto BHs in a cyclic scenario of the universe described by DGP and RSII branes. Our motivation is to find out if there is any remanent mass of BH when it undergoes a turnaround in a cyclic scenario. Babichev et al [15] has shown that BH mass decrease with phantom energy accretion on it. Hence the BH will disappear before the turnaround in an oscillating universe. But Sun [16] provided a mechanism which showed that in an universe described by modified Friedmann cosmology the destruction of BHs is totally out of question, as there is always a remanent mass of a BH facing a turnaround. In this paper our motivation is to testify the above fact for brane-world cosmology and find out the fate of a BH undergoing phantom energy accretion in an oscillating universe.

The paper is organised as follows: In section 2 we discuss the mechanism of cyclic universe in RSII brane model. Section 3 deals with an identical mechanism for DGP brane model. In section 4, we present an argument regarding Hawking evaporation of remanent BHs. Finally the paper ends with some concluding remarks in section 5.

2 Cyclic Universe in RSII Brane Model

The novel feature of the RS models compared to previous higher-dimensional models is that the observable 3 dimensions are protected from the large extra dimension (at low energies) by curvature rather than straightforward compactification. In RS II model the effective equations of motion on the 3-brane embedded in 5D bulk having Z_2 -symmetry are given by [17, 18, 1, 19, 20, 21]

$${}^{(4)}G_{\mu\nu} = -\Lambda_4 q_{\mu\nu} + \kappa_4^2 \tau_{\mu\nu} + \kappa_5^4 \Pi_{\mu\nu} - E_{\mu\nu} \quad (1)$$

where

$$\kappa_4^2 = \frac{1}{6} \lambda \kappa_5^4, \quad (2)$$

$$\Lambda_4 = \frac{1}{2} \kappa_5^2 \left(\Lambda_5 + \frac{1}{6} \kappa_5^2 \lambda^2 \right) \quad (3)$$

and

$$\Pi_{\mu\nu} = -\frac{1}{4} \tau_{\mu\alpha} \tau_{\nu}^{\alpha} + \frac{1}{12} \tau \tau_{\mu\nu} + \frac{1}{8} q_{\mu\nu} \tau_{\alpha\beta} \tau^{\alpha\beta} - \frac{1}{24} q_{\mu\nu} \tau^2 \quad (4)$$

and $E_{\mu\nu}$ is the electric part of the 5D Weyl tensor. Here κ_5 , Λ_5 , $\tau_{\mu\nu}$ and Λ_4 are respectively the 5D gravitational coupling constant, 5D cosmological constant, the brane tension (vacuum energy), brane energy-momentum tensor and effective 4D cosmological constant. The explicit form of the above modified Einstein equations in flat universe for RSII brane are

$$3H^2 = \Lambda_4 + \kappa_4^2 \rho + \frac{\kappa_4^2}{2\lambda} \rho^2 + \frac{6}{\lambda \kappa_4^2} \mathcal{U} \quad (5)$$

and

$$2\dot{H} + 3H^2 = \Lambda_4 - \kappa_4^2 p - \frac{\kappa_4^2}{2\lambda} \rho p - \frac{\kappa_4^2}{2\lambda} \rho^2 - \frac{2}{\lambda \kappa_4^2} \mathcal{U} \quad (6)$$

The dark radiation \mathcal{U} obeys

$$\dot{\mathcal{U}} + 4H\mathcal{U} = 0 \quad (7)$$

where ρ and p are the total energy density and pressure respectively.

2.1 Phantom Energy Accretion

We consider an homogeneous and isotropic universe filled with DE fluid, with DE density ρ and pressure p . For an asymptotic observer the black hole mass, M changes at the rate of [15]

$$\dot{M} = 4\pi A M^2 (\rho + p) \quad (8)$$

Here the overdot denotes the derivative with respect to cosmic time. Moreover it has been considered that $G = c = 1$. We consider an universe dominated by DE. The Friedmann equation for the expanding universe is given by

$$H^2 = \frac{8\pi}{3} \rho \quad (9)$$

Where $H \equiv \frac{\dot{a}}{a}$ is the Hubble parameter. The conservation equation for DE is given by

$$\dot{\rho} + 3H(\rho + p) = 0 \quad (10)$$

Phantom DE has the equation of state $w = \frac{p}{\rho} < -1$. We get that $\rho \propto a^{-3(1+w)}$, which shows that ρ increases with the expansion of the universe.

In this section we will investigate the dynamics of cyclic universe in RSII brane model. The modified Friedmann equation for RSII brane model is given by equation (5). Putting $\mathcal{U} = 0$, the equation becomes,

$$H^2 = \frac{\kappa_4^2}{3} \rho \left(1 + \frac{\rho}{2\lambda}\right) + \frac{\Lambda_4}{3} \quad (11)$$

In the expanding phase of the universe, the phantom energy density, ρ increases. From the above equation we see that at the turnaround the phantom energy density is given by,

$$\rho_c = -\lambda \pm \sqrt{\lambda^2 - \alpha} \quad (12)$$

where $\alpha = \frac{2\lambda\Lambda_4}{\kappa_4^2}$.

From the above value we see that when $\lambda > 0$, then $\Lambda_4 < 0$, and when $\lambda < 0$, then $\Lambda_4 > 0$. $\lambda > \frac{2\Lambda_4}{\kappa_4^2}$. This shows that either the brane tension or the bulk cosmological constant has to be negative, so that the universe undergoes a bounce, and a possibility for the cyclic scenario is evident. Here ρ_c stands for critical density. After the turnaround the universe begins to contract. The reason behind this being, that in the contracting phase the non-phantom components of the universe increase and begins to dominate the evolution. In this phase, again when the dominant energy density reaches the critical value given by equation (12), a bounce occurs. Thus we get an oscillating scenario. We intend to study the variation of BH mass with phantom energy accretion around it, in this oscillating cosmological model.

2.2 Scenario Before Turnaround

Using equations (8) and (10) we get

$$\frac{dM}{M^2} = -\frac{4\pi A}{3H} d\rho \quad (13)$$

Before turnaround, we have

$$H = \frac{\kappa_4}{\sqrt{3}} \sqrt{\rho \left(1 + \frac{\rho}{2\lambda}\right) + \frac{\Lambda_4}{\kappa_4^2}} \quad (14)$$

Now substituting the above value of H in equation (13) we get,

$$\frac{dM}{M^2} = -\frac{D}{\sqrt{\rho \left(1 + \frac{\rho}{2\lambda}\right) + \frac{\Lambda_4}{\kappa_4^2}}} d\rho \quad (15)$$

where $D = \frac{4\pi A}{\sqrt{3}\kappa_4}$ and ρ_c is given by equation (12). Now integrating equation (15) we get,

$$M = \frac{M_i}{1 + DM_i \sqrt{2\lambda} \log \left(\frac{\lambda + \rho + \sqrt{\alpha + 2\lambda\rho + \rho^2}}{\lambda + \rho_i + \sqrt{\alpha + 2\lambda\rho_i + \rho_i^2}} \right)} \quad (16)$$

Here ρ_i and M_i denotes respectively the phantom energy density and the black hole mass at the moment when the phantom energy density just begins to dominate the evolution of the universe. In general, $\rho_i \ll \rho_c$ and $\rho_i \leq \rho \leq \rho_c$. So using equation (12) we obtain $\frac{\rho_i}{\rho_c} \rightarrow 0$. Hence from equation (16), we obtain

$$M \simeq \frac{M_i}{1 + DM_i \sqrt{2\lambda} \log \left(\frac{\lambda + \rho + \sqrt{\alpha + 2\lambda\rho + \rho^2}}{\lambda} \right)} \quad (17)$$

At the turnaround, $\rho = \rho_c$ and hence the black hole mass at the turnaround becomes,

$$M_c \simeq \frac{M_i}{1 + DM_i \sqrt{2\lambda} \log \left(\frac{\lambda + \rho_c + \sqrt{\alpha + 2\lambda\rho_c + \rho_c^2}}{\lambda} \right)} \quad (18)$$

This shows that there is a remnant mass of the BH when the turnaround occurs. Hence this result is different from the result obtained by Zhang [22]. It is quite clear from the above equations that initially, through phantom energy accretion, the BH mass decreases, until it reaches the minimum value M_c at the turnaround in the expanding phase. For $M_i \gg M_p = G^{-\frac{1}{2}}$, M_c becomes independent of M_i

$$M_c \simeq \frac{1}{D\sqrt{2\lambda} \log \left(\frac{\lambda + \rho_c + \sqrt{\alpha + 2\lambda\rho_c + \rho_c^2}}{\lambda} \right)} \quad (19)$$

2.3 Scenario After Turnaround

After the turnaround as expected the the universe will contract and consequently the phantom energy density ρ starts decreasing. During the contraction it is obvious that the time derivative of the scale factor will become negative, and as a result H becomes negative. So, here we take the value of H as,

$$H = -\frac{\kappa_4}{\sqrt{3}} \sqrt{\rho \left(1 + \frac{\rho}{2\lambda} \right) + \frac{\Lambda_4}{\kappa_4^2}} \quad (20)$$

Substituting the above value of H in equation (13) we get,

$$\frac{dM}{M^2} = \frac{D}{\sqrt{\rho \left(1 + \frac{\rho}{2\lambda} \right) + \frac{\Lambda_4}{\kappa_4^2}}} d\rho \quad (21)$$

Now integrating the above equation we get,

$$M = \frac{M_c}{1 + DM_c \sqrt{2\lambda} \log \left(\frac{\lambda + \rho_c + \sqrt{\alpha + 2\lambda\rho_c + \rho_c^2}}{\lambda + \rho + \sqrt{\alpha + 2\lambda\rho + \rho^2}} \right)} \quad (22)$$

where $\rho \leq \rho_c$. The above equation shows that as the universe contracts, the BH mass continue to decrease. When $\rho \ll \rho_c$, then the BH mass is,

$$M_f \simeq \frac{M_c}{1 + DM_c \sqrt{2\lambda} \log \left(\frac{\lambda + \rho_c + \sqrt{\alpha + 2\lambda\rho_c + \rho_c^2}}{\lambda} \right)} \quad (23)$$

For $M_i \gg M_p$, using equation (19) we find that the final mass of BHs is

$$M_f \simeq \frac{M_c}{2} \simeq \frac{1}{2D\sqrt{2\lambda} \log \left(\frac{\lambda + \rho_c + \sqrt{\alpha + 2\lambda\rho_c + \rho_c^2}}{\lambda} \right)} \quad (24)$$

Hence M_f is independent of M_i .

Variation of BH mass in the expanding cycle(before turnaround)

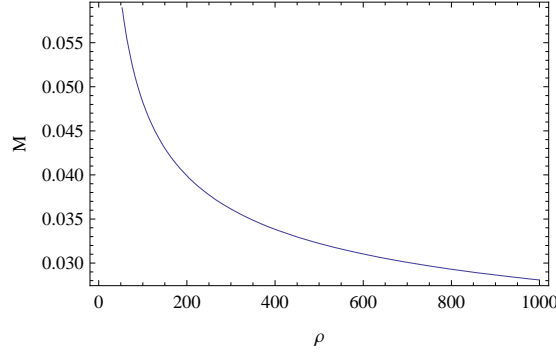


Fig. 1

Variation of BH mass in the contracting cycle(after turnaround)

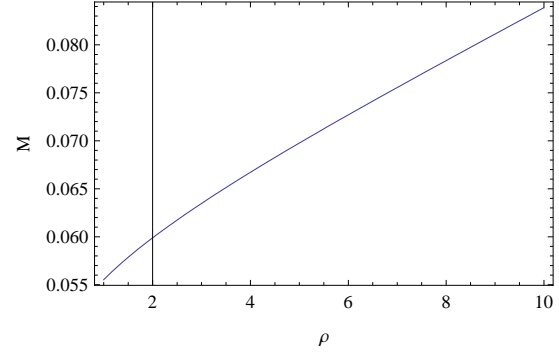


Fig. 2

Fig 1 : The mass of the black hole is plotted against the increasing density of phantom dark energy before turnaround. Other parameters are fixed at $\alpha = -5$, $\lambda = 10$, $D = 1.5$, $M_i = 10000$, $\rho_i = 0.0001$

Fig 2 : The mass of the black hole is plotted against the decreasing density of phantom dark energy after turnaround. Other parameters are fixed at $\alpha = -5$, $\lambda = 10$, $D = 1.5$, $M_c = 100$, $\rho_c = 100$

3 Cyclic Universe in DGP Brane Model

While flat, homogeneous and isotropic brane is being considered, the Friedmann equation in DGP brane model is modified to the equation

$$H^2 = \left(\sqrt{\frac{\rho}{3} + \frac{1}{4r_c^2}} + \epsilon \frac{1}{2r_c} \right)^2 \quad (25)$$

where $H = \frac{\dot{a}}{a}$ is the Hubble parameter, ρ is the total cosmic fluid energy density and $r_c = \frac{M_p^2}{2M_5^2}$ is the cross-over scale which determines the transition from 4D to 5D behaviour and $\epsilon = \pm 1$ (choosing $M_p^2 = 8\pi G = 1$). For $\epsilon = +1$, we have standard DGP(+) model which is self accelerating model without any form of DE, and effective w is always non-phantom. However for $\epsilon = -1$, we have DGP(-) model which does not self accelerate but requires DE on the brane. It experiences 5D gravitational modifications to its dynamics which effectively screen DE.

Like the original Friedmann equation, and contrary to RSII brane equation, the DGP brane equation does not readily support an oscillating universe undergoing turnarounds. This fact is quite obvious from the equation (25). We can see that there is no possibility of turnaround. Therefore we will propose a modified DGP brane equation, that can support a cyclic universe. Sun et al in [23] proposed a method, by which they were able to modify the Friedmann equation into a form that avoids the Big Rip singularity and gives a bouncing cosmological model. Their investigations led to the fact that the character of physics changes remarkably near the Planck scale. We know that a de Sitter universe with a cosmological constant, Λ is similar to a BH. Also it has a temperature, $T \sim H$. Hence by conjecturing physics at the Planck scale, they actually modified the definition of Hawking temperature, and subsequently obtained the modified Friedmann equation. Sun in [16] used this modified equation successfully to study the phantom energy accretion on BHs. Hence, taking a leaf out of their book, we proceed to modify the DGP brane equation as follows: The proposed modified DGP brane equation is

$$H^2 = \left(\sqrt{\frac{\rho}{3} + \frac{1}{4r_c^2}} + \epsilon \frac{1}{2r_c} \right)^2 \left(1 - \frac{\rho}{\rho'_c} \right) \quad (26)$$

From the above equation, we see that the turnaround occurs for $\rho = \rho'_c$.

3.1 Scenario Before Turnaround

The expression for H for the expanding phase is given by,

$$H = \left(\sqrt{\frac{\rho}{3} + \frac{1}{4r_c^2}} + \epsilon \frac{1}{2r_c} \right) \sqrt{\left(1 - \frac{\rho}{\rho'_c} \right)} \quad (27)$$

Using equation (13) and (27) we get,

$$\frac{dM}{M^2} = - \frac{D'}{\left(\sqrt{\frac{\rho}{3} + \frac{1}{4r_c^2}} + \frac{\epsilon}{2r_c} \right) \sqrt{1 - \frac{\rho}{\rho'_c}}} \quad (28)$$

where $D' = \frac{4\pi A}{3}$. Now integrating the above equation we get,

$$\begin{aligned} \frac{1}{M} = \frac{1}{M_i} + D' & \left[\sqrt{3\rho'_c} \left(\arctan \left(\frac{3A - \rho'_c + 2\rho}{2\sqrt{(\rho'_c - \rho)(3A - \rho)}} \right) - \arctan \left(\frac{3A - \rho'_c + 2\rho_i}{2\sqrt{(\rho'_c - \rho_i)(3A - \rho_i)}} \right) \right) \right. \\ & + \frac{3B\sqrt{\rho'_c}}{\sqrt{3B^2 - 3A - \rho'_c}} \left(\arctan \left(\frac{9A^2 + 3B^2(\rho'_c - 2\rho) + \rho\rho'_c + 3A(\rho + \rho'_c - 3B^2)}{2B\sqrt{3\rho'_c(3A + \rho)(3B^2 - 3A - \rho'_c)} \left(1 - \frac{\rho}{\rho'_c} \right)} \right) \right. \\ & \quad \left. - \arctan \left(\frac{9A^2 + 3B^2(\rho'_c - 2\rho_i) + \rho_i\rho'_c + 3A(\rho_i + \rho'_c - 3B^2)}{2B\sqrt{3\rho'_c(3A + \rho_i)(3B^2 - 3A - \rho'_c)} \left(1 - \frac{\rho_i}{\rho'_c} \right)} \right) \right) \\ & \left. + \frac{6B\sqrt{\rho'_c}}{\sqrt{3A - 3B^2 + \rho'_c}} \left(\arctan \left(\frac{\sqrt{\rho'_c \left(1 - \frac{\rho}{\rho'_c} \right)}}{\sqrt{3A - 3B^2 + \rho'_c}} \right) - \arctan \left(\frac{\sqrt{\rho'_c \left(1 - \frac{\rho_i}{\rho'_c} \right)}}{\sqrt{3A - 3B^2 + \rho'_c}} \right) \right) \right] \quad (29) \end{aligned}$$

Where $A = \frac{1}{4r_c^2}$, and $B = \frac{\epsilon}{2r_c}$

Now as in the case of RSII brane, $\rho_i \ll \rho'_c$, which implies that $\frac{\rho_i}{\rho'_c} \rightarrow 0$. Hence from the above equation we obtain,

$$\begin{aligned} \frac{1}{M} \simeq \frac{1}{M_i} + D' & \left[\sqrt{3\rho'_c} \left(\arctan \left(\frac{3A - \rho'_c + 2\rho}{2\sqrt{(\rho'_c - \rho)(3A - \rho)}} \right) - \arctan \left(\frac{3A - \rho'_c}{2\sqrt{3A\rho'_c}} \right) \right) \right. \\ & + \frac{3B\sqrt{\rho'_c}}{\sqrt{3B^2 - 3A - \rho'_c}} \left(\arctan \left(\frac{9A^2 + 3B^2(\rho'_c - 2\rho) + \rho\rho'_c + 3A(\rho + \rho'_c - 3B^2)}{2B\sqrt{3\rho'_c(3A + \rho)(3B^2 - 3A - \rho'_c)} \left(1 - \frac{\rho}{\rho'_c} \right)} \right) \right. \\ & \quad \left. - \arctan \left(\frac{9A^2 + 3B^2\rho'_c + 3A(\rho'_c - 3B^2)}{2B\sqrt{9A\rho'_c(3B^2 - 3A - \rho'_c)}} \right) \right) \\ & \left. + \frac{6B\sqrt{\rho'_c}}{\sqrt{3A - 3B^2 + \rho'_c}} \left(\arctan \left(\frac{\sqrt{\rho'_c \left(1 - \frac{\rho}{\rho'_c} \right)}}{\sqrt{3A - 3B^2 + \rho'_c}} \right) - \arctan \left(\frac{\sqrt{\rho'_c}}{\sqrt{3A - 3B^2 + \rho'_c}} \right) \right) \right] \quad (30) \end{aligned}$$

At the turnaround $\rho = \rho'_c$. Hence we get,

$$\frac{1}{M_c} \simeq \frac{1}{M_i} + D' \left[\frac{\pi}{2} \sqrt{3\rho'_c} - \arctan \left(\frac{3A - \rho'_c}{2\sqrt{3A\rho'_c}} \right) \right]$$

$$\begin{aligned}
& + \frac{3B\sqrt{\rho'_c}}{\sqrt{3B^2 - 3A - \rho'_c}} \left(\frac{\pi}{2} - \arctan \left(\frac{9A^2 + 3B^2\rho'_c + 3A(\rho'_c - 3B^2)}{2B\sqrt{9A\rho'_c(3B^2 - 3A - \rho'_c)}} \right) \right) \\
& - \frac{6B\sqrt{\rho'_c}}{\sqrt{3A - 3B^2 + \rho'_c}} \arctan \left(\frac{\sqrt{\rho'_c}}{\sqrt{3A - 3B^2 + \rho'_c}} \right) \Big] \tag{31}
\end{aligned}$$

The above equation gives the remanent mass of the BH at the turnaround. Hence we see that the mass of BH gradually decrease during the expanding phase of cyclic universe, due to phantom energy accretion and finally becomes minimum at the turnaround, when it is called the critical mass, given by equation(30). For $M_i \gg M_p$, M_c becomes independent of M_i and is given by the following relation,

$$\begin{aligned}
\frac{1}{M_c} & \simeq D' \left[\frac{\pi}{2} \sqrt{3\rho'_c} - \arctan \left(\frac{3A - \rho'_c}{2\sqrt{3A\rho'_c}} \right) \right. \\
& + \frac{3B\sqrt{\rho'_c}}{\sqrt{3B^2 - 3A - \rho'_c}} \left(\frac{\pi}{2} - \arctan \left(\frac{9A^2 + 3B^2\rho'_c + 3A(\rho'_c - 3B^2)}{2B\sqrt{9A\rho'_c(3B^2 - 3A - \rho'_c)}} \right) \right) \\
& \left. - \frac{6B\sqrt{\rho'_c}}{\sqrt{3A - 3B^2 + \rho'_c}} \arctan \left(\frac{\sqrt{\rho'_c}}{\sqrt{3A - 3B^2 + \rho'_c}} \right) \right] \tag{32}
\end{aligned}$$

3.2 Scenario After Turnaround

Just like the RSII model, in DGP model as well, the universe starts to contract after turnaround, and the non-phantom components starts to dominate the evolution. The expression for H is given by,

$$H = - \left(\sqrt{\frac{\rho}{3} + \frac{1}{4r_c^2}} + \epsilon \frac{1}{2r_c} \right) \sqrt{\left(1 - \frac{\rho}{\rho'_c} \right)} \tag{33}$$

Using equation (13) and (33) we get,

$$\frac{dM}{M^2} = \frac{D'}{\left(\sqrt{\frac{\rho}{3} + \frac{1}{4r_c^2}} + \frac{\epsilon}{2r_c} \right) \sqrt{1 - \frac{\rho}{\rho'_c}}} \tag{34}$$

Integrating the above equation we get,

$$\begin{aligned}
\frac{1}{M} & = -\frac{1}{M_i} - D' \left[\sqrt{3\rho'_c} \left(\arctan \left(\frac{3A - \rho'_c + 2\rho}{2\sqrt{(\rho'_c - \rho)(3A - \rho)}} \right) - \arctan \left(\frac{3A - \rho'_c + 2\rho_i}{2\sqrt{(\rho'_c - \rho_i)(3A - \rho_i)}} \right) \right) \right. \\
& + \frac{3B\sqrt{\rho'_c}}{\sqrt{3B^2 - 3A - \rho'_c}} \left(\arctan \left(\frac{9A^2 + 3B^2(\rho'_c - 2\rho) + \rho\rho'_c + 3A(\rho + \rho'_c - 3B^2)}{2B\sqrt{3\rho'_c(3A + \rho)(3B^2 - 3A - \rho'_c)} \left(1 - \frac{\rho}{\rho'_c} \right)} \right) \right. \\
& \left. \left. - \arctan \left(\frac{9A^2 + 3B^2(\rho'_c - 2\rho_i) + \rho_i\rho'_c + 3A(\rho_i + \rho'_c - 3B^2)}{2B\sqrt{3\rho'_c(3A + \rho_i)(3B^2 - 3A - \rho'_c)} \left(1 - \frac{\rho_i}{\rho'_c} \right)} \right) \right) \right. \\
& \left. + \frac{6B\sqrt{\rho'_c}}{\sqrt{3A - 3B^2 + \rho'_c}} \left(\arctan \left(\frac{\sqrt{\rho'_c} \left(1 - \frac{\rho}{\rho'_c} \right)}{\sqrt{3A - 3B^2 + \rho'_c}} \right) - \arctan \left(\frac{\sqrt{\rho'_c} \left(1 - \frac{\rho_i}{\rho'_c} \right)}{\sqrt{3A - 3B^2 + \rho'_c}} \right) \right) \right] \tag{35}
\end{aligned}$$

Now as in the previous section, $\rho_i \ll \rho'_c$, which implies that $\frac{\rho_i}{\rho'_c} \rightarrow 0$. Hence from the above equation we obtain,

$$\begin{aligned}
\frac{1}{M} \simeq & -\frac{1}{M_i} - D' \left[\sqrt{3\rho'_c} \left(\arctan \left(\frac{3A - \rho'_c + 2\rho}{2\sqrt{(\rho'_c - \rho)(3A - \rho)}} \right) - \arctan \left(\frac{3A - \rho'_c}{2\sqrt{3A\rho'_c}} \right) \right) \right. \\
& + \frac{3B\sqrt{\rho'_c}}{\sqrt{3B^2 - 3A - \rho'_c}} \left(\arctan \left(\frac{9A^2 + 3B^2(\rho'_c - 2\rho) + \rho\rho'_c + 3A(\rho + \rho'_c - 3B^2)}{2B\sqrt{3\rho'_c(3A + \rho)(3B^2 - 3A - \rho'_c)} \left(1 - \frac{\rho}{\rho'_c}\right)} \right) \right. \\
& \quad \left. \left. - \arctan \left(\frac{9A^2 + 3B^2\rho'_c + 3A(\rho'_c - 3B^2)}{2B\sqrt{9A\rho'_c(3B^2 - 3A - \rho'_c)}} \right) \right) \right. \\
& \left. + \frac{6B\sqrt{\rho'_c}}{\sqrt{3A - 3B^2 + \rho'_c}} \left(\arctan \left(\frac{\sqrt{\rho'_c \left(1 - \frac{\rho}{\rho'_c}\right)}}{\sqrt{3A - 3B^2 + \rho'_c}} \right) - \arctan \left(\frac{\sqrt{\rho'_c}}{\sqrt{3A - 3B^2 + \rho'_c}} \right) \right) \right] \quad (36)
\end{aligned}$$

At the turnaround $\rho = \rho'_c$. Hence we get,

$$\begin{aligned}
\frac{1}{M_c} \simeq & -\frac{1}{M_i} - D' \left[\frac{\pi}{2} \sqrt{3\rho'_c} - \arctan \left(\frac{3A - \rho'_c}{2\sqrt{3A\rho'_c}} \right) \right. \\
& + \frac{3B\sqrt{\rho'_c}}{\sqrt{3B^2 - 3A - \rho'_c}} \left(\frac{\pi}{2} - \arctan \left(\frac{9A^2 + 3B^2\rho'_c + 3A(\rho'_c - 3B^2)}{2B\sqrt{9A\rho'_c(3B^2 - 3A - \rho'_c)}} \right) \right) \\
& \left. - \frac{6B\sqrt{\rho'_c}}{\sqrt{3A - 3B^2 + \rho'_c}} \arctan \left(\frac{\sqrt{\rho'_c}}{\sqrt{3A - 3B^2 + \rho'_c}} \right) \right] \quad (37)
\end{aligned}$$

As before the above equation gives the remanent mass of the BH at the turnaround. For $M_i \gg M_p$, M_c becomes independent of M_i and is given by the following relation,

$$\begin{aligned}
\frac{1}{M_c} \simeq & D' \left[\arctan \left(\frac{3A - \rho'_c}{2\sqrt{3A\rho'_c}} \right) \right. \\
& - \frac{3B\sqrt{\rho'_c}}{\sqrt{3B^2 - 3A - \rho'_c}} \left(\frac{\pi}{2} - \arctan \left(\frac{9A^2 + 3B^2\rho'_c + 3A(\rho'_c - 3B^2)}{2B\sqrt{9A\rho'_c(3B^2 - 3A - \rho'_c)}} \right) \right) \\
& \left. + \frac{6B\sqrt{\rho'_c}}{\sqrt{3A - 3B^2 + \rho'_c}} \arctan \left(\frac{\sqrt{\rho'_c}}{\sqrt{3A - 3B^2 + \rho'_c}} \right) - \frac{\pi}{2} \sqrt{3\rho'_c} \right] \quad (38)
\end{aligned}$$

4 Hawking evaporation of black holes

From the above evaluation it is clear that there is always a remanent mass of the BHs at the turnaround in an oscillating universe. So contrary to existing literature we conclude that there is no possibility of destruction of BH. The BHs formed during the expanding cycle of the cyclic universe survive into the next cycle and eventually grow in size. Therefore they create undesired non-uniformity in a nearly uniform universe. Eventually the BHs will occupy the entire volume of the horizon and will be responsible for the destruction of the cyclic models. This is a serious defect indeed! Hence the problem posed by the BHs in a cyclic universe still stands un-eliminated.

But in [24] it has been argued that for a BH with mass $M = 10^5 M_p$, Hawking evaporation takes place in time $\tau \sim \frac{25\pi M^3}{M_p^4} \sim 10^{-27}$ sec and ultimately the BH becomes non-existent, thus causing no problems. Here M_p represents Planck mass. In our above calculations we have considered $G = M_p^{-2} = 1$. Rewriting equation (18), for RSII brane we get,

$$M_c \simeq \frac{M_i}{1 + \frac{DM_i \sqrt{2\lambda} \log \left(\frac{\lambda + \rho_c + \sqrt{\alpha + 2\lambda\rho_c + \rho_c^2}}{\lambda} \right)}{M_p^3}} \quad (39)$$

D is taken as a constant of the order unity and $\log\left(\frac{\lambda+\rho_c+\sqrt{\alpha+2\lambda\rho_c+\rho_c^2}}{\lambda}\right) \sim M_p^2$. Then we have $M_c \sim \frac{M_i}{1+\frac{M_i}{M_p}}$. For a BH with $M_i \gg M_p$, $M_c \sim M_p$, i.e., the remanent mass of the BH is of the order of Planck mass. Hence the remanent BH undergoes Hawking evaporation in time $\tau \sim 10^{-43}$ sec at the order of Planck time. A similar evaluation is possible for DGP brane model as well. So fortunately, the remanent BHs do not cause any problems. This gives a possible solution of the BH problem in cyclic universe described by brane-world scenario.

5 Conclusion and Discussion

A serious problem is posed by the existence of black holes in an oscillating universe. In [24] Brown et al suggested that in an oscillating cosmology the black holes keep losing mass due to phantom energy accretion before totally disappearing before the turnaround. Babichev et al in [15] devised a successful mechanism which was in accordance with the result given by Brown et al. In this paper we have investigated the outcome of phantom energy accretion on black holes in a cyclic universe described by brane-worlds. It is seen that RSII brane model readily incorporates the oscillating nature in its framework. The only condition being the negativity of the brane tension, λ or the bulk cosmological constant, Λ_4 . But unlike RSII brane model, DGP brane does not readily support the cyclic nature of the universe. So a modified DGP brane equation, that supports cyclic turnarounds has been proposed for our evaluation.

It is found that for RSII brane model, during the expanding phase the black hole mass gradually decrease with the increase of phantom energy density and finally reaches a critical value at which the turnaround occurs. This result is absolutely consistent with the known results in literature. In the contracting phase the black hole mass again decreases with the increase in the non-phantom components of the universe. This is however contrary to our expectations. So it is understood that our evaluations after the turnaround are not really rigid. We see that the black holes in a cyclic universe reaches a remanent mass, M_c before turnaround. So, the remanent mass implies that the destruction of black holes is not a real possibility in the cyclic cosmology, with phantom energy turnarounds, for a universe characterized by brane gravity. However fortunately we find that the remanent masses of black holes at turnaround do not cause problems. The reason being that these remanent black holes Hawking evaporate in a time $\tau \sim 10^{-43}$.

Acknowledgement:

The author acknowledges Mr.Ritabrata Biswas and Dr.Ujjal Debnath for helpful discussions.

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